UNIT - 4

VECTOR DIFFERENCIATION

Vector:-

It is a quantity having both magnitude and direction, Eg:- Velocity, acceleration.

Scalar:-

It is a quantity having magnitude but no direction. Eg:- Distance, force, time.

Null Vector:- A vector whose magnitude is zero is called null vector.

Unit Vector:- A vector whose magnitue is one is called unit vector.

Vector function:- A function is in the form of $\bar{i} f_1(t) + \bar{j} f_2(t) + \bar{k} f_3(t)$ is called vector function and denoted by $\bar{f}(t)$. where f_1, f_2, f_3 are scalar function.

Derivative of a vector function :-

Let $\bar{f}$ be a vector function on an interval 'I' and $a \in I$. Then,

$$\underset{t \to a}{Lt} \frac{\bar{f}(t) - \bar{f}(a)}{t - a}$$ If exist's is ~~at~~

called derivative of $\bar{f}'(a)$ or $\left[\frac{d\bar{f}}{dt}\right]_{t=a}$.

Properties of Differenciable function:

- Derivative of constant $= \bar{0}$.
- $\frac{d}{dt}(\bar{a} \pm \bar{b}) = \frac{d}{dt}\bar{a} \pm \frac{d}{dt}(\bar{b})$
- $\frac{d}{dt}(\bar{a} \cdot \bar{b}) = \frac{d\bar{a}}{dt} \cdot \bar{b} + \bar{a} \cdot \frac{d\bar{b}}{dt}$
- $\frac{d}{dt}(\bar{a} \times \bar{b}) = \frac{d\bar{a}}{dt} \times \bar{b} + \bar{a} \frac{d\bar{b}}{dt}$
- If $\bar{f}$ is differenciable vector and ϕ is scalar differenciable function, Then

$$\frac{d}{dt}(\phi \bar{f}) = \phi \frac{d\bar{f}}{dt} + \bar{f} \frac{d\phi}{dt}$$

Vector Differenciable Operator :-

The vector differenciable operator (∇) Dul (Dell) (nebbla) is defined as $\boxed{\nabla = \bar{i}\frac{\partial}{\partial x} + \bar{j}\frac{\partial}{\partial y} + \bar{k}\frac{\partial}{\partial z}.}$

Gradiant of a Scalar point function :-

Let $\phi(x, y, z)$ be a scalar point function defined in some region of space. Then the vector function $\bar{i}\frac{\partial \phi}{\partial x} + \bar{j}\frac{\partial \phi}{\partial y} + \bar{k}\frac{\partial \phi}{\partial z}$ is Known as gradient of ϕ. (or) grad ϕ (or) $\nabla\phi$

$$\boxed{\nabla\phi = \bar{i}\frac{\partial \phi}{\partial x} + \bar{j}\frac{\partial \phi}{\partial y} + \bar{k}\frac{\partial \phi}{\partial z}}$$

- Properties :

- If "f" and "g" are two scalar functions Then $\text{grad}(f \pm g) = \text{grad}(f) \pm \text{grad}(g)$

- The necessary and sufficient condition for a scalar point function to be constant. Then

$$\text{grad} f \text{ (or) } \nabla f = 0.$$

- $\text{grad}(fg) = g(\text{grad} f) + f(\text{grad } g)$

- $\text{grad}(f/g) = \frac{g(\text{grad} f) - f(\text{grad} g)}{g^2}.$

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· If c = contat $\left[f' = \frac{df}{dx} \right]$

$$\text{grad}\,(cf) = c\,(\text{grad}\, f).$$

· Directional Derivative:

The directional derivative of a scalar point function "ϕ" at a point $p(x, y, z)$ in the direction of unit vector $(\bar{e})$ then

$$\boxed{\bar{e} \cdot \nabla \phi}$$ where $\bar{e}$ is unit vector.

note:- The unit vector of a vector is

$$\bar{e} = \frac{\bar{a}}{|\bar{a}|}.$$

· Unit Vector for a Scalar:

$$\boxed{\bar{e} = \frac{\nabla f}{|\nabla f|}}$$

· Angle between 2 surfaces:-

$$\cos\theta = \frac{\nabla f \cdot \nabla g}{|\nabla f|\,|\nabla g|}$$

Greates Value of directional derivatie (ϕ) at 'p' point:

$$P = |\text{grad}\, \phi|$$

$\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$

$$\frac{dz}{dH}$$

$|\bar{r}| = \sqrt{x^2+y^2+z^2}$

$r^2 = x^2+y^2+z^2.$

$$\frac{\partial r}{\partial x} = \frac{x}{r}, \quad \frac{\partial r}{\partial y} = \frac{y}{r}, \quad \frac{\partial r}{\partial z} = \frac{z}{r}.$$

19 Show that $\nabla f(r) = \frac{f'(r)}{r} \bar{r}$

$\bar{r} = x\bar{i} + y\bar{k} + z\bar{k}$

$|\bar{r}| = \sqrt{x^2+y^2+z^2}$

$r^2 = x^2+y^2+z^2$

$$\frac{\partial r}{\partial x} = \frac{x}{r}, \quad \frac{\partial r}{\partial y} = \frac{y}{r}, \quad \frac{\partial r}{\partial z} = \frac{z}{r}$$

$LHS = \nabla f(r)$

$$= \bar{i}\frac{df(r)}{dx} + \bar{j}\frac{df(r)}{dy} + \bar{k}\frac{df(r)}{dz}$$

$$= \cancel{\bar{i}\frac{x}{r} + \bar{j}\frac{y}{r} + \bar{k}\frac{z}{r}}$$

$$= \bar{i} f'(r)\frac{dr}{dx} + \bar{j} f'(r)\frac{dr}{dy} + \bar{k} f'(r)\frac{dr}{dz}$$

$$= \bar{i} f'(r)\frac{x}{r} + \bar{j} f'(r) + \frac{y}{r} + \bar{k} f'(r)\frac{z}{r}$$

$$= \frac{f'(r)}{r}(\bar{i}+\bar{j}+\bar{k})$$

$$= \frac{\bar{f}(r)}{r} \cdot \bar{r} = RHS \quad \text{Hence Proved}$$

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2Q Show that ~~$\frac{d(2y)}{dx} = \frac{dy}{dx}\left(\frac{d\,2y}{dx}\right)$~~

$$\nabla \cdot r^n = n r^{n-2} \cdot \bar{r}$$

$$\bar{r} = \bar{i}x + \bar{j}y + \bar{k}y$$

$$r^2 = x^2 + y^2 + z^2$$

$$|r| = \sqrt{x^2+y^2+z^2}$$

$$\frac{dr}{dx} = \frac{x}{r}, \quad \frac{dr}{dy} = \frac{y}{r}, \quad \frac{dr}{dz} = \frac{z}{r},$$

$$LHS = \nabla r^n = \left(\bar{i}\frac{d}{dx} \; \bar{j}\frac{d}{dy} \; \bar{k}\frac{d}{dz}\right) r^n$$

$$= \bar{i}\frac{dr}{dx}(n)r^{n-1} + \bar{j}\frac{dr}{dy}(n)r^{n-1} + \bar{k}\frac{dr}{dz}(n)r^{n-1}$$

$$= n\,r^{n-1}\left(\bar{i}\frac{x}{r} + \bar{j}\frac{y}{r} + \bar{k}\frac{z}{r}\right)$$

$$= \frac{n\,r^{n-1}}{r}(\bar{i} + \bar{j} + \bar{k})$$

$$LHS = \frac{n\,r^{n-2}\,\bar{r}}{r} = RHS$$

Hence proved.

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3Q) $\nabla(x^2+y^2z)$

$$\left(i\frac{d}{dx}+j\frac{d}{dy}+k\frac{d}{dz}\right)(x^2+y^2z)$$

$$2x\left(\hat{i}\frac{dx}{dx}\right)+\left(\hat{j}\frac{d\,2yz\,y}{dy}\right)+\left(\hat{k}\frac{dz}{dz}\right)y^2$$

$$=\hat{i}(2x)+\hat{j}(2yz)+\hat{k}(y^2)$$

4Q) Find the unit normal vector to the given surface $x^2y+2xz=4$, at the point $(2,-2,3)$

$4(-2)+2(2)(3)$

$12-8=4$.

$$\Rightarrow f=x^2y+2xz-4.$$

$$\bar{e}=\frac{\nabla f}{|\nabla f|},\ \bar{e}=\frac{\bar{a}}{|\bar{a}|}$$

$$\Rightarrow \nabla f=\left(i\frac{d}{dx}+j\frac{d}{dy}+k\frac{d}{dz}\right)x^2y+2xz-4$$

$$=\cancel{2x(\hat{i})y+2y(\hat{i})+x^2(\hat{j})+2x(\hat{j})}+\hat{k}(0)$$

$$\cancel{=(2xy+2y)\hat{i}+x^2+2x(\hat{j})}$$

$$\nabla f=\hat{i}(2xy+2z)+\hat{j}(x^2)+\hat{k}(2x)$$

$$=-2\hat{i}+4\hat{j}+4\hat{k}$$

$$|\nabla f|=\sqrt{(2xy+2z)^2+x^4+(2x)^2}=\cancel{2\sqrt{3}}\ 6$$

$2,-2,3$

$$\bar{e}=\frac{-2\hat{i}+4\hat{j}+4\hat{k}}{6}$$

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Q) Find the unit normal vector to the surface

$z = x^2+y^2$ at $(-1,-2,5)$

$f = x^2+y^2-z$

Sol:-

$\nabla f = 2x(\bar{i}) + 2y(\bar{j}) - \bar{k}$

at $(-1,-2,5)$

$\nabla f = -2\bar{i} - 4\bar{j} - \bar{k}$

$\bar{e} = \dfrac{\nabla f}{|\nabla f|}$

$|\nabla f| = \sqrt{4+16+1} = \sqrt{21}$

$$\bar{e} = \frac{-2\bar{i}-4\bar{j}-\bar{k}}{\sqrt{21}}$$

Q) Find the unit normal vector to the surface

$x^2y + 2xz^2 = 8$ at $(1,0,2)$

Solution :- $f = x^2y+2xz^2-8$

$\nabla f = (2xy+2z^2)\hat{i} + x^2\hat{j} + 2zx(2)\hat{k}$

$(1,0,2)$

$\nabla f = (2(1)(0)+2(2)^2)\hat{i} + 1^2\hat{j} + 2(2)(1)(2)\hat{k}$

$\nabla f = 8\hat{i}+\hat{j}+8\hat{k}$

$|\nabla f| = \sqrt{129}$

$$\boxed{\bar{e} = \frac{8\bar{i}+\bar{j}+8\bar{k}}{\sqrt{129}}}$$

Find the angle between Surfaces $x^2+y^2+z^2=9$ at $(2,-1,2)$ $z=x^2+y^2-3$

$f = x^2+y^2+z^2-9$

$\nabla f = 2x(\vec{i}) + 2y(\vec{j}) + 2z(\vec{k})$

$\nabla f = 4\vec{i} - 2\vec{j} + 4\vec{k}.$

$|\nabla f| = 6$

$g = x^2+y^2-z-3$

$\nabla g = 2x\,\vec{i} + 2y\,\hat{j} - \vec{k}$

$4\vec{i} - 2\vec{j} - \vec{k}$

$|\nabla g| = \cancel{3}.\ \sqrt{21}$

$$\cos\theta = \frac{(4\vec{i} - 2\vec{j} + 4\vec{k}).(4\vec{i} - 2\vec{j} - \vec{k})}{6\ \sqrt{21}}$$

$$\cos\theta = \cancel{\frac{10\vec{i} + 20\vec{j}}{6\sqrt{21}}} = \frac{16}{6\sqrt{21}}$$

$$\theta = \cos^{-1}\left(\frac{16}{6\sqrt{21}}\right) = 54.4^\circ.$$

$\therefore$ angle between the surfaces is 54.4° at $(2,-1,2)$

Find the angle between the sphere $x^2+y^2+z^2=29$ and $x^2+y^2+z^2+4x-6y-8z-47=0$, at $(4,-3,2)$

$\nabla f =$

$(2x+4)\hat{i} + (2y-6)\hat{j} + (2z-8)\vec{k}$

$12\vec{i} - 12\vec{j} - 4\vec{k}$

$\nabla g =$

$2x\,\hat{i} + 2y\,\hat{j} + 2z\,\vec{k}$

$8\hat{i} - 6\vec{j} + 4\vec{k}$

$\cos\theta = \frac{\nabla f \nabla g}{|\nabla f||\nabla g|} \Rightarrow \cos\theta = \frac{152}{4\sqrt{19} \times 2\sqrt{29}}$ $\sqrt{116}$ $(\sqrt{304})$

$\theta = 35.95^\circ$

Find the angle between surfaces

$x^2 = yz$ at $(1,1,1)$ and $(2,4,1)$

$f = x^2 - yz$

$\nabla f = 2x\hat{i} - z\hat{j} - y\hat{z}$.

at $(1,1,1)$. at $(2,4,1)$

$\nabla f_1 = 2i - j - z$; $\nabla f_2 = 4i - 4j - k$

$|\nabla f_1| = \sqrt{6}$ $|\nabla f| = \sqrt{33}$

$\nabla f_1 \cdot \nabla f_2 = 13$

$\cos\theta = \dfrac{13}{\sqrt{6}\sqrt{33}}$ $= 22.5^\circ$.

31/07/2023

1) $x^2yz + 4xz^2$ at $(1,-2,-1)$ $\Rightarrow \phi$

directional derivative

direction $2\bar{i} - \bar{j} - 2\bar{k}$. $\Rightarrow \bar{e} = \dfrac{2\bar{i} - \bar{j} - 2\bar{k}}{\sqrt{4+1+4}}$

$= \dfrac{2\bar{i} - \bar{j} - 2\bar{k}}{3}$

$\nabla\, x^2yz + 4xz^2$.

$(2xyz)\hat{i} + 4z^2\hat{i}$

$x^2z\hat{j} + (z^2y + 2zx)\hat{k}$

$(2xyz + 4z^2)\hat{i} + (x^2z)\hat{j} + (x^2y + 8zx)\hat{k}$.

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$(2(1)(-2)(4) + 4(-1)^2)\hat{i} + 1^2(-1)\hat{j}$

$+(1^2(-2) + 2(-1)(1))\hat{k}$.

$+4 + 4\hat{i} + -1\hat{j} + -2-2\hat{k}$

$8\hat{i} - \hat{j} - 40\hat{k} \Rightarrow \nabla\phi$

$\bar{c} = \dfrac{2\bar{i} - \hat{j} - 2\hat{k}}{3}$

$\bar{e} \cdot \nabla\phi = \dfrac{2\bar{i} - \bar{k} - 2\bar{k}}{3} \cdot 8\bar{i} - \bar{j} - 10\bar{k}$.

$= \cancel{37}/3$

$= 12.33$ 4.

2 find the directional derivative of $xyz^2 + xz$ at 1,1,1 in direction of normal to the surface $3xy^2 + y = z$. at $(0,1,1)$

$\phi = xyz^2 + xz$

$\nabla\phi \quad (yz^2+z)\hat{i} + xz^2\hat{j} + (2z\,xy + x)\hat{k}$

$\nabla\phi$ at $(1,1,1)$ $\qquad \bar{e} = \dfrac{\bar{j}+\bar{k}}{\sqrt{2}} = \dfrac{\nabla f}{|\nabla f|}$

$= 2\bar{i} + 1\bar{j} + 3\bar{k}$

$\nabla f = \dfrac{3i + 1j - k}{\sqrt{11}}$

$= \dfrac{4}{\sqrt{11}} = \dfrac{4}{\sqrt{4}}$

3) find the directional derivative of

$f = x^2 - y^2 + 2z^2$ at $P(1,2,3)$ in direction of line PQ, $Q = (5,0,4)$

$PQ = 4x - 2y + z$

$x + 2y + 3z$

$= 5x + 4z$

$4x - 2y + z$

$$\bar{n} = \frac{\nabla f}{|\nabla f|} = \frac{2x\,\hat{i} + 2y\,\hat{j} + 4z\,\hat{k}}{}$$

$$= \frac{2\hat{i} - 4\hat{j} + 12\hat{k}}{2\sqrt{41}} \qquad \sqrt{164}$$

$$\bar{e} = \frac{PQ}{|PQ|} = \frac{4i - 2j + k}{\sqrt{16+4+1}} = \frac{28}{\cancel{\sqrt{164}}\sqrt{21}}$$

$$= \cancel{\frac{14}{\sqrt{41}}}\ \frac{28}{\sqrt{21}}$$

$$\bar{e} \cdot \nabla f = \frac{28}{\sqrt{21}}$$

4) directional derivative of the function $xy^2 + yz^2 + zx^2$ along the tangent to the curve $x = t$, $y = t^2$, $z = t^3$ at $(1,1,1)$

$$\bar{r} = ti + t^2 j + t^3 k$$

$$\frac{dr}{dt} = i + 2j + 3k$$

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① In what direction from the point $(-1,1,2)$ is the directional derivative of $\phi = xy^2z^3$ a maximum. What is the magnitude of the max.

Sol^n $\phi = xy^2z^3$

$$\nabla\phi = y^2z^3\,\hat{i} + 2xyz^3\,j + 3xy^2z^2\,k$$

at $(-1,1,2)$.

$$\nabla\phi = (1)^2(2)^3\,i + 2(-1)(1)(2)^3\,j + 3(-1)(1)^2(2)^2\,k$$

$$\nabla\phi = 8i - 16j - 12k.$$

$$\nabla\phi = 8i-16j-12k$$

$$\cancel{|\nabla\phi|} = \frac{\cancel{8i-16j-12k}}{\cancel{4\sqrt{29}}} = |\nabla\phi| = \sqrt{8^2+16^2+12^2} = 4\sqrt{29}$$

IMPORTANT

2. Find a, b.

surfaces, $ax^2 - byz = (a+2)x$ and $4x^2y + z^3 = 4$ may intersect orthogonally at $(1,-1,2)$

$$\nabla f \cdot \nabla g = 0$$

$$ax^2 - byz - (a+2)x = 4x^2y + z^3 - 4$$

$$a(1)^2 - b(-1)(2) - (a+2)(1) = 4(1)^2(-1) + 2^3 - 4$$

$$a + 2b - a - 2 = -4 - 4 + 8$$

$$2b = 2$$

$$b = 1.$$

$\nabla f = 2ax + (a+2)\,i - bz\bar{j} - by\bar{k}$.

$= (\cancel{3a^2+2})\,i - b2j + bk$
$\quad a-2$

$\nabla g = 8xy\,\bar{i} + 4x^2\bar{j} + 3z^2\bar{k}$

$= -8\bar{i} + 4\bar{j} + 12\bar{k}$

$\theta = \nabla f \cdot \nabla g = (\overset{a-2}{\cancel{3a+4}})(-8) + (-b2)(4) + 12(b) = 0$

$\therefore b = 1$

$(\overset{(a-2)}{\cancel{3a+4}})(-8) + (-8) + (-12) = 0$

$\cancel{-3a - 32 - 8 - 12 = 0}$

$\cancel{a = -\frac{52}{3}} = \cancel{a = 0.5}$

$\nabla f = (a-2)\bar{i} - 2\bar{j} + k$

$\nabla g = -8i + 4j + 12k$

$(a-2)(-8) + (-2)(4) + (1)(12) = 0$

$8a = 16 - 8 + 12$

$a = 20/8 = 2.5$

3. Findthe constants a, b such that the surfaces

$5x^2 - 2yz - 9x = 0 \qquad ax^2y + bz^3 = 4.$

$5 - 2(-2) - 9$ cut orthogonally at $(1,-1,2)$

$0 =$ $\boxed{a = +12.}$ $b = 3/2$. $-2a + a$

$a(1)^2(-1) + b(2)^3 = 4$

$2ay \quad b(8)$

$-1 \quad 10x - 9 \quad ax^2$

$-4 \quad \cancel{+9} \quad -2z$

$3 \quad +2 \quad -2y \quad b(3)z^2$

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divergence of Vector :-

Let $\bar{f}$ be a ~~conb~~ divergent of a vector point function. Then,

$\bar{i} \cdot \frac{\partial \bar{f}}{dx} + \bar{j} \cdot \frac{\partial \bar{f}}{dy} + \bar{k} \frac{\partial \bar{f}}{\partial z}$ is divergence of $\bar{f}$

and represented as div $\bar{f}$ i.e,

$$\text{div } \bar{f} = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{dz}.$$

Solinoidal Vector :-

A vector point function is called to be Solinoidal if div $\bar{f} = 0$.

③ find div $\bar{f}$ where $\bar{f}$ = grad $(x^3 + y^3 + z^3 - 3xyz)$

$\nabla f = \bar{f} = (3x^2 - 3yz)\bar{i} + (3y^2 - 3xz)\bar{j} + (3z^2 - 3xy)\bar{k}$

div $\bar{f}$ = $6x\,\bar{i} + 6y\,\bar{j} + 6z\,\bar{k}$

div $\bar{f}$ = $6x + 6y + 6z$

④ if $\bar{f} = (x + 3y)\bar{i} + (y - 2z)\bar{j} + (x + Pz)\bar{k}$ Solinoidal

div $\bar{f} = 0 = 1\bar{i} + 1\bar{j} + P\bar{z} = 0$. $P = -2$..

$5x^2 - 2yz - 9x = 0$

16

~ 20

12

~ 8

$(10x - 9)i - (2z)j - (2y)k = 0$ ~ 20

$2axyi + ax^2 j + 3z^2 b = 0$

$(1, -1, 2)$

$(10-9)i - 2(2)j - 2(-1)k = 0$

$(-1)2(1)a + a(1)^2 + b(3)(2)^2 = 0$

$1 - 4 + 2 = -2a + a + 12b$

$3a + 4b = -1$

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- Find div $\bar{f}$ where $f = r^n \bar{r}$. find if it is solinaidal.

(or)

- Prove that $r^n \bar{r}$ is solinaidal if $n = -3$.

(or)

- P.T div $(r^n \bar{r}) = (n+3)r^n$. S.T $\bar{r}/r^3$ is solonaidal

$\boxed{\text{Soln}}$ $\bar{f} = r^n \bar{r} = r^n(x\hat{i} + y\hat{j} + z\hat{k})$

$r^2 = x^2 + y^2 + z^2$

$\frac{\partial r}{\partial x} = \frac{x}{r}, \quad \frac{\partial r}{\partial y} = \frac{y}{r}, \quad \frac{\partial r}{\partial z} = \frac{z}{r}$

$\cancel{\text{div} f = r^n + r^n + r^n = 3r^n}$

$\cancel{x \cdot n r^{n-1}} \cancel{+ y \frac{\partial r}{\partial x}}$

$$\operatorname{div} \bar{f} = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}$$

$$= \frac{\partial}{\partial x}(x r^n) + \frac{\partial}{\partial y}(y r^n) + \frac{\partial}{\partial z}(z r^n)$$

$$= x n r^{n-1} \frac{\partial r}{\partial x} + r^n + y n r^{n-1} \frac{\partial r}{\partial y} + r^n + z n r^{n-1} \frac{\partial r}{\partial z} + r^n$$

$$(n r^{n-1}) \left(x \frac{\partial r}{\partial x} + y \frac{\partial r}{\partial y} + z \frac{\partial r}{\partial z} \right) + 3 r^n$$

$$= n r^{n-1} \left(x \left(\frac{x}{r}\right) + y \left(\frac{y}{r}\right) + z \left(\frac{z}{r}\right) \right) + 3 r^n$$

$$= n r^{n-2} (x^2 + y^2 + z^2) + 3 r^n$$

$$= n r^{n-2} (r^2) + 3 r^n$$

$$= n r^n + 3 r^n$$

$$(n+3) r^n.$$

i) $\therefore \operatorname{div} \bar{f} = (n+3) r^n$

It is not Solivoidal. Vector

except for $n = -3$).

ii) If $n = -3$

n

$(n+3) r^n$

$(3+3) r^{-3}$

$= 0$

$\therefore$ It is Solivoidal.

iii) a) $\operatorname{div}(r^n \bar{r}) = (n+3) r^n$ Hence Proved.

v) $\frac{\bar{r}}{r^3} = \bar{r} r^{-3} \Rightarrow n = -3$, for $\bar{r} r^n$

for $n = -3$.

div $\bar{f}$ is Salinoidal Vector.

Let $\bar{f}$ be a continusly differenciable vector paint function. Then the vector function defined by $\bar{i} \times \frac{\partial \bar{f}}{\partial x} + \bar{j} \times \frac{\partial \bar{f}}{\partial y} + \bar{k} \times \frac{\partial \bar{f}}{\partial z}$ is called curl of $\bar{f}$.:

It is denoted by $\nabla \times \bar{f}$ (or) Curl $\bar{f}$

i.e, $$\text{Curl } \bar{f} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_1 & f_2 & f_3 \end{vmatrix}$$

(or)

$$\text{curl } \bar{f} = \sum \bar{i} \times \frac{\partial \bar{f}}{\partial x}$$

$\nabla \cdot \bar{f} = \text{div } \bar{f}$

$\nabla \bar{f} = \text{grad } \bar{f}$

$\nabla \times \bar{f} = \text{Curl } \bar{f}$

Irrotational of Vector –

A vect. $\bar{f}$ is said to be irrotational if

Curl $\bar{f} = 0$.

(1) Prove that $\bar{F} = yz\bar{i} + zx\bar{j} + xy\bar{k}$ is irrotational.

Solution –

$$\text{Curl } \bar{f} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ yz & zx & xy \end{vmatrix}$$

$$i\left[\frac{d}{dy}(xy) - \frac{d}{dz}(zx)\right] - j\left[\frac{d}{dx}\,xy - \frac{d}{dz}(yz)\right] + z\left[\frac{d}{dx}(zx) - \frac{d}{dy}(yz)\right]$$

$$= i[x - x] + j[y - y] + k[z - z]$$

$= 0.$

$\therefore$ Curl $\bar{f} = 0$

Hence It is irrotational.

(2) If $\bar{f} = x\bar{i} - y^2\bar{j} + z^3\bar{k}$. find Curl $\bar{f}$.

$$i\left[\frac{d}{dy}(z^3) - \frac{d}{dz}(-y^2)\right] - j\left[\frac{d}{dx}(z^3) - \frac{d}{dz}(x)\right] + k\left[\frac{d}{dx}(-y^2) - \frac{d}{dy}(x)\right]$$

$= i[0] + j[0] + k[0] = 0 \therefore$ Curl $\bar{f} = 0.$

③ If $\bar{f} = xy^2\, i + 2x^2yz\, j - 3yz^2\, k$

$$\text{curl}\, \bar{f} = i\left[-3z^3 - 2x^2y\right] + j\left[0 - 0\right] + k\left[4xyz - 2xy\right]$$

at $(1, -1, 1)$

$$\left[-3(1)^3 - 2(1)^2(-1)\right] i + j(0) + \left[4(1)(-1)(1) - 2(1)(-1)\right] k$$

$$(-3+2)\, i + 0\, j + (-4+2)\, k$$

$$= \cancel{-2\bar{i} - 6\bar{k}} \quad -\bar{i} - 2\bar{k}$$

④ find Curl $\bar{f}$ where $\bar{f} = \text{grad}\,(x^3 + y^3 + z^3 - 3xyz)$

$$\bar{f} = (3x^2 - 3yz)\,\bar{i} + (3y^2 - 3xz)\,\bar{j} + (3z^3 - 3xy)\,\bar{k}$$

$$\text{Curl}\, \bar{f} = i\left[(-3x) - (-3x)\right] + j\left[(-3y) - (-3y)\right] + k\left[(-3z) - (-3z)\right]$$

$$= i\,[0] + j\,[0] + k\,[0]$$

$\therefore$ Curl $\bar{f} = 0$